Assessing Student Conceptual Understanding of Force and Motion with Model Analysis

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ABSTRACT

This study aims to assess student conceptual model of understanding about force and motion by employing a new analysis method, called "model analysis". This method was established from qualitative researches in order to qualitatively represent a framework of student understanding. With model analysis, we can obtain students' alternative knowledge and the probabilities for students to use such knowledge in a range of equivalent contexts. The model analysis consists of two algorithms—concentration factor and model estimation. This paper only presents results from using the model estimation algorithm.

In order to use the model analysis efficiently, the data must be collected from a well-designed multiple-choice test. The Force and Motion Conceptual Evaluation (FMCE), the most well-known test for probing mechanics conceptual understanding was administered to 746 engineering freshmen taking an introductory physics with calculus at Chiang Mai University. Only 545 complete student responses were analyzed by the model analysis.

The class model density matrices for both pre/post scores were constructed. In order to determine characteristics of the pre/post class, eigenvalue decomposition was used to analyze both matrices. Each matrix had a large eigenvalue (> 0.65), indicating the dominant features of the single-student model vectors. This model eigenvectors well represented the overall model structure of pre/post class. Then the pre/post class model states were characterized by a class model point on a model plot. Both pre/post points were located in the incorrect model region, so both pre/post class states were still in a misconception state. However, there was a small shift of post-class model point towards the correct model, indicating a small improvement of overall understanding.

Key words: Model Analysis, FMCE, Conceptual Understanding, Force and Motion

INTRODUCTION

Over three decades, results from physics education research (PER) indicate that most students come to a physics classroom with misconceptions, originating from their misinterpretations of everyday's experience and previous instruction (McDermott and Redish, 1999). These misconceptions affect how students respond to instruction, so physics instructors should acquire student prior understandings in order to design more-effective teaching methods. In PER, free-response questions, interviews and multiple-choice questions are often used to probe student understandings.

In a large-scale setting, multiple-choice test is the easiest to analyze and the cheapest to conduct, but there is a lack of efficient methods to analyze student responses. Many physics education researchers have developed various multiple-choice conceptual tests to detect student misconceptions such as Force Concept Inventory (FCI) (Hestenes et al., 1992) and Force and Motion Conceptual Evaluation (FMCE) (Thornton and Sokoloff, 1998). Many studies employed these instruments to measure student understanding, however, the results from these instruments tend to be used to obtain overall scores and average pre/post gains (Hake, 1998; Huffman, 1998; Savinainen and Philip, 2002; Bonham et al., 2003).

The typical analysis of the multiple-choice tests not only fails to provide information about students' misconceptions but also ignores students' wrong answers containing a large amount of valuable information. Thus, the model analysis was developed to extract information about models of student understanding from their responses to multiple-choice test (Bao and Redish, 2006). This method is most effective in detecting well-defined misconceptions. These misconceptions were documented from qualitative researches that students often enter a classroom with a few number of strong naïve conceptions. These misconceptions are often in conflict with or encourage misinterpretations of the expert view. One of the well-defined misconceptions is about force and motion. Therefore, this study aims to investigate students' conceptions of force and motion by using the model analysis.

METHODOLOGY

Settings

The data were collected during the first semester of academic year 2006. The FMCE pre- and post-tests were given to students taking physics for engineering and agro-industrial students I (PHYS207105) on the first and the last day of class. Students were given 45 minutes to complete the FMCE. There were 746 students taking both pre- and post-test. After disregarding incomplete responses, only data from 545 students were analyzed by the model analysis.

Instrument

The FMCE was developed by Thornton and Sokoloff (1998). Since then it has become one of the most popular instruments used to probe student understanding of Newtonian mechanics. This test consists of 47 multiple-choice single-response items and one open-ended question. FMCE items can be categorized into five clusters of basic mechanics concepts—velocity, acceleration, Newton's first and second laws, Newton's third law and energy. For each item of FMCE, if students think that none of the choice is correct, they can choose to answer choice J. The Thai version of FMCE translated by Physics Education Network of Thailand (PENThai) was used to collect data.

TRADITIONAL ANALYSIS

The traditional and typical quantitative analysis of multiple-choice test was used in order to compare the results with the one using model analysis. Student responses were input into an excel template designed by Wittmann (2001), then the percentage of students' correct responses on pre-and post-test were plotted according to different clusters and overall scores, as shown in Figure 1. The percentage gain is calculated as follows:

$$\% \, Gain \quad = \quad \frac{\% \, post - \% \, pre}{(100 - \% \, pre)} \qquad (1)$$

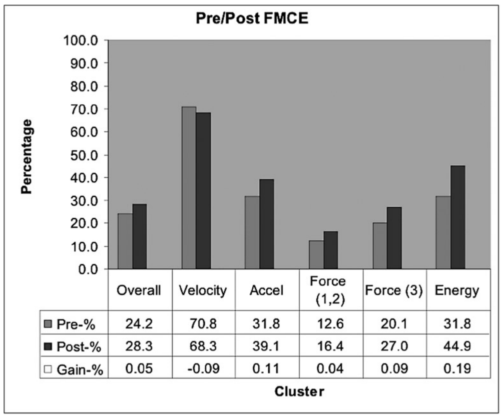


Figure 1: Percentage of Pre/Post FMCE scores categorized into overall and five concept clusters.

From Figure 1, Newton's 1st and 2nd laws cluster has the lowest percent correction in both pre/post test scores. Therefore, using the model analysis should provide useful results of student's understanding model in this topic.

MODEL ANALYSIS

The model analysis consists of two algorithms—concentration factor and model estimation. The concentration factor shows a distribution of student responses, whether they are clustered on certain choices or scattered among all choices. The model estimation is a quantitative evaluation of student models of understandings derived from a numerical analysis of student responses on multiple-choice tests. Since the concentration factor of student responses on FMCE has already been reported (Wattanakasiwich, 2006), this paper only presents the result from using the model estimation algorithm.

Theoretical Framework: Student Model State

It is a continuing effort among educational researchers to look for new ways to understand student learning process. However, learning is a complicated process, so it could not be measured directly. We can only model student ways of thinking and further improve our understanding of student learning. From physics teaching experiences, students are not consistent in solving problems and sometimes even use contradictory ideas to answer similar questions. In many cases when a similar concept is presented under different physical contexts, students may have difficulties in identifying the correct physics. They tend to use pieces of knowledge that are induced by the surface features of the specific contexts. Therefore, students seem to function as if they hold a mixture of different models (a correct one and incorrect ones) without knowing the appropriate situation in which to apply them.

From results of cognitive research, it may be of great interest to consider the student as always being in a consistent mental state. For students to reach a complete expert model, they need to go through a process of conceptual change, as shown in Figure 2. The mixed state is regarded as an important transitional stage for a student to reach a complete favorable conceptual change in learning physics. Hence, measurements of such mixed states have important values in assessment and instruction (Bao and Redish, 2006). Students in this mixed model state (sometimes referred to as a hybrid model) often combine certain parts of the new knowledge and parts of their existing knowledge. It is a solution of reconciliation to produce a locally-consistent model for two types of knowledge which are otherwise contradictory.

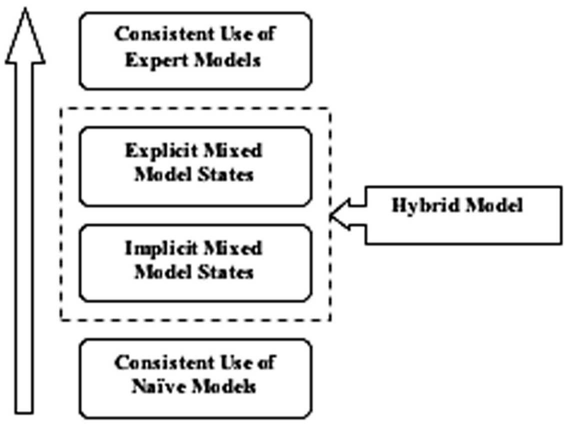


Figure 2: A process of model development leading to a conceptual change.

When students answer a particular physics question, the context of that question triggers them to apply a certain model. The process of model activation is complicated, so we cannot measure students' model states directly. Bao and Redish

(2006) proposed that the probability of activating model state could be obtained by analyzing students' responses to a well-designed instrument. In other words, the process of model activation could be treated as a process of quantum measurement. Accordingly, the mental state of the student can be represented with respect to a set of common models in a linear vector space, referred to as *the model space*. Each common model is associated with an element of an orthonormal basis, e_n as shown in Figure 3. This supports the fact that different mental models can have similar features. Bao and Redish (2006) indicated that studies in neuroscience about neural networks stimulated the ideas of using this representation.

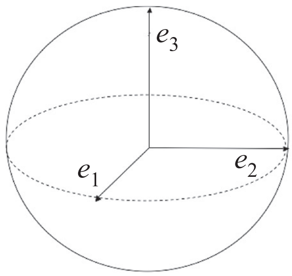


Figure 3: A model space consisting of three orthogonal model vectors—$\boldsymbol{e}_1$, $\boldsymbol{e}_2$ and $\boldsymbol{e}_3$.

Construct Student Model State

In order to construct student model state, student responses from a well-designed multiple-choice test are required. The "well-designed" instrument has to be developed so that the choices of the question are designed to probe the different common student models. These models have been revealed from qualitative physics education research. There are 2 common models of force and motion found from PER and a null model refers to other ideas or incomplete answers:

Model 1: It is necessary to have a force to maintain motion and there is no such thing as a "force in the direction of motion." (Correct)

Model 2: A force is needed to maintain motion. This model also includes the ideas that there is always a force in the direction of motion and that the force is directly related to the velocity of motion. (Incorrect)

Model 3: Null model

In the FMCE, four questions (questions 2, 5, 11 and 12) are associated with Newton's first and second law. When analyzing each question, we can identify distracters associating with a particular model, as shown in Table 1.

Table 1: Distracters of force-motion questions with a specific model.

Question	Model 1	Model 2	Model 3	The kth student
2	D	B	Others	A
5	D	B	Others	B
11	A	G	Others	G
12	A	D	Others	A

The last column represents choices that an arbitrary kth student might answer for each question. From this student's answers, we can construct the kth student's model state or u_k, as in (2), where m is number of questions associated with the specific concept, and n_1, n_2, n_3 are numbers of choices corresponding with Model 1, Model 2 and Model 3, respectively (Bao and Redish, 2006).

$$u_k = \frac{1}{\sqrt{m}}\begin{bmatrix}\sqrt{n_1^k}\\ \sqrt{n_2^k}\\ \sqrt{n_3^k}\end{bmatrix} \Rightarrow u_k = \frac{1}{2}\begin{bmatrix}1\\ \sqrt{2}\\ 1\end{bmatrix} \tag{2}$$

Construct Class Model Density Matrix

After constructing the kth student's model state, his model density matrix can be determined as below:

$$D_k = u_k \otimes u_k^T = \frac{1}{m}\begin{bmatrix} n_1^k & \sqrt{n_1^k n_2^k} & \sqrt{n_1^k n_3^k}\\ \sqrt{n_2^k n_1^k} & n_{21}^k & \sqrt{n_2^k n_3^k}\\ \sqrt{n_3^k n_1^k} & \sqrt{n_3^k n_2^k} & n_3^k \end{bmatrix} \tag{3}$$

Then each student's model density matrix is summed up to get a class model density matrix as in (4), where is total number of students in the class.

$$D = \frac{1}{N}\sum_{k=1}^{N} D_k \tag{4}$$

After calculating the class model density matrix for both pre- and post-test, eigenvalues and eigenvectors for both matrices were calculated by using MATLAB program, as shown in Table 2.

Table 2: The pre/post class model density matrix, their eigenvalues and their eigenvectors.

	Pre-			Post-		
Class model density matrix	$\begin{bmatrix} 0.11 & 0.12 & 0.05 \\ 0.12 & 0.71 & 0.22 \\ 0.22 & 0.22 & 0.18 \end{bmatrix}$			$\begin{bmatrix} 0.14 & 0.13 & 0.06 \\ 0.13 & 0.68 & 0.19 \\ 0.19 & 0.19 & 0.17 \end{bmatrix}$		
Eigenvalues	0.82	0.05	0.13	0.79	0.06	0.15
Eigenvectors	$\begin{bmatrix} -0.18 \\ -0.91 \\ -0.37 \end{bmatrix}$	$\begin{bmatrix} -0.29 \\ -0.26 \\ 0.92 \end{bmatrix}$	$\begin{bmatrix} 0.13 \\ -0.38 \\ 0.92 \end{bmatrix}$	$\begin{bmatrix} -0.22 \\ -0.91 \\ -0.35 \end{bmatrix}$	$\begin{bmatrix} -0.34 \\ -0.21 \\ 0.92 \end{bmatrix}$	$\begin{bmatrix} 0.31 \\ -0.39 \\ 0.87 \end{bmatrix}$

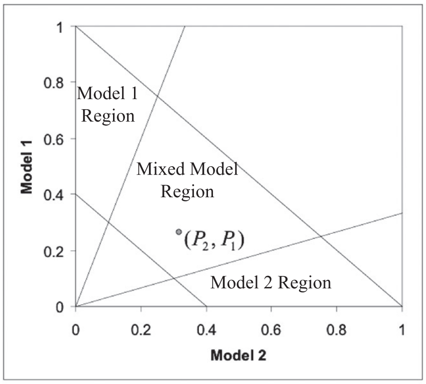
The characteristic of models that students in the class used can be obtained by analyzing these matrices. Both the similarity of individual student's model vectors and the number of students with similar model vectors influence the eigenvalue, especially if the class model density matrix has a dominant and large eigenvalue (> 0.65). This implies that most students in the class have similar model vectors, referred to as *a primary vector* (Bao and Redish, 2006). From Table 2, both pre/post matrices have dominant eigenvalues, so both classes have primary vectors which give a good evaluation of the model structure of both classes. Bao and Redish (2006) suggested to use a model plot to further analyze the primary vector.

Model Plot

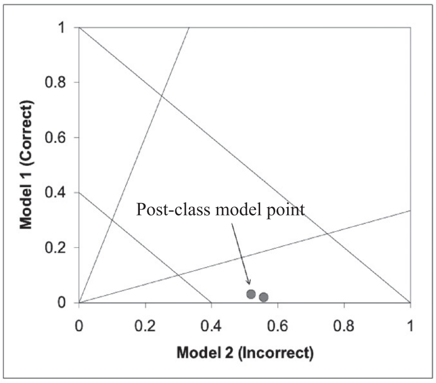
The model plot is for representing the class model state with respect to common models. The model plot, in this case, is a two-dimensional graph because there are two common models used in answering the force and motion questions. The class model states or eigenvectors with dominant eigenvalues can be represented as a class model point on the model plot with a coordinate (P_1, P_2). From Table 3, vertical and horizontal components for pre/post class model point are calculated and plotted on the model plot as shown in Figure 4.

Table 3: The pre/post dominant eigenvalues, class model eigenvectors and vertical/horizontal components for the class model point.

	Pre-	Post-
Eigenvalues	0.82	0.79
Eigenvectors	$\begin{bmatrix} -0.18 \\ -0.91 \\ -0.37 \end{bmatrix}$	$\begin{bmatrix} -0.22 \\ -0.91 \\ -0.35 \end{bmatrix}$
Model Point		
A vertical component	$P_1=(0.82)^2(-0.18)^2=0.02$	$P_1=(0.79)^2(-0.22)^2=0.03$
A horizontal component	$P_2=(0.82)^2(-0.91)^2=0.56$	$P_2=(0.79)^2(-0.91)^2=0.52$



(a)



(b)

Figure 4: (a) Meaning of some regions in the model plot.
(b) Model plot comparing between pre- and post-class model points.

From the model plot in Figure 4 (b), the post-class model point is located in the Model 2 region. This indicates that most students in the class still had a misconception about force and motion. Due to the model analysis, the characteristics of this misconception are known, so an instructor can use this information to improve teaching of this class. A small shift of post-class model point towards the correct model indicates a small improvement of overall class understanding in this topic.

CONCLUSION

In this paper, the author used the model analysis to analyze quantitative data in order to obtain a qualitative result of student conceptual understanding of force and motion. This method is useful in analyzing student's knowledge states in large classes with well-designed multiple-choice questions. With the measurement data from FMCE, a single-student model state can be created. This state represents student probabilities in applying the different common models. Then the individual student model states are summed up over the class to create the class model density matrix. Using the eigenvalue decomposition, the eigenvalues and eigenvectors of the density matrix are obtained, and these give information about the state of the class's knowledge. The class model point is obtained and plotted on the model plot. This is helpful in providing clear information about class model state. From the result of this study, only small progress in understanding was found by comparing the pre/post class model points. However, the class model state was still in a misconception region even after a proper instruction. This information is extremely helpful in notifying the instructor and can be used to improve a future instruction.

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