Strong Comultiplication Modules

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ABSTRACT

In this paper, we introduced the concepts of strong comultiplication modules and copure submodules and some related results more obtained.

Key Words: Comultiplication modules, Strong comultiplication modules, Copure submodules

INTRODUCTION

Throughout this paper, R will denote a commutative ring with identity and □ will denote the ring of integers. The dual notion of multiplication modules was introduced by Ansari-Toroghy and Farshadifar (2007) and the first properties of this class of modules had been considered. We recall that M is a comultiplication module (Ansari-Toroghy and Farshadifar, 2007) if for every submodule N of M there exists an ideal I of R such that $N = (0:_M I)$. Also, it is shown that (Ansari-Toroghy and Farshadifar, 2007, 3.7) M is a *comultiplication module* if and only if for each submodule N of M, $N = (0\ _M Ann_R(N))$. Let M be an R-module. In Section 3 of this paper, we will introduce the concepts of strong comultiplication modules and copure submodules. M is said to be a *strong comultiplication module* if M is a comultiplication R-module which satisfies the double annihilator conditions (see 1.1 (d)). Furthermore, a submodule N of M is said to be *copure* if $(N:_M I) = N + (0:_M I)$ for each ideal I of R. Now let M be an R-module and let N be a submodule of M. Among the other results, it is shown (see 2.5) that whenever M is a strong comultiplication module, M / N is a comultiplication R-module if and only if $Ann_R (N)\ Ann_R (K / N) = Ann_R (K)$ for each submodule K of M with $N \subseteq K$ Moreover, it is shown (see 2.12) that pure and copure submodules of M are the same over a principal ideal domain. Also it is proved (see 2.13) that whenever M is a strong comultiplication module, N is a copure submodule of M if and only if $Ann_R (N)$ is a pure ideal of R. Moreover, it is shown (see 2.13) that if N is a copure submodule of a strong comultiplication module M, then $(N:_R M) = Ann_R\ Ann_R (N)$ and $Ann_R (N)$ is the intersection of all ideals I of R such that $N = (N:_M I)$. Finally, it is proved (see 2.15) that if M is a comultiplication (resp. multiplication) module such that $Soc(M)$ (resp. $Rad(M)$) is a pure (resp. copure) submodule of M, then $M = Soc(M)$ (resp. $Rad(M)$=0).