On the Weak Solution of the Compound Ultra-hyperbolic Equation

Amnuay Kananthai* and Kamsing Nonlaopon

Department of Mathematics, Faculty of Science, Chiang Mai University, Chiangmai 50200, Thailand

**Corresponding author. E-mail: malamnka@science.cmu.ac.th*

ABSTRACT

In this paper we have studied the compound ultra-hyperbolic equation of the form

$$\sum_{r=0}^{m} c_r \Box^r u(x) = f(x),$$

where $\Box^r$ is the ultra-hyperbolic operator iterated r-times ($r = 0, 1, 2, ..., m$), f is a given generalized function, u is an unknown function, $x = (x_1, x_2, ..., x_n) \in \mathbb{R}^n$ the Euclidean n-dimensional spaces and c_r is a constant.

It is found that the equation above has a weak solution u(x) which is of the form Marcel Riesz's kernel and moreover, such a solution is unique.

1. INTRODUCTION

Consider the equation

$$\Box^k u(x) = f(x), \tag{1.1}$$

where u and f are some generalized functions, and $\Box^k$ is the ultra-hyperbolic operator iterated k -times and is defined by

$$\Box^k = \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + ... + \frac{\partial^2}{\partial x^2_p} - \frac{\partial^2}{\partial x^2_{p+1}} - \frac{\partial^2}{\partial x^2_{p+2}} - ... - \frac{\partial^2}{\partial x^2_{p+q}} \right)^k, \tag{1.2}$$

$p + q = n$ is the dimension of the space $\mathbb{R}^n$, $x = (x_1, x_2, ..., x_n) \in \mathbb{R}^n$, and k is a nonnegative integer.

Trione (1987) has shown that (1.1) has $u(x) = R_{2k}(x)$ as a unique elementary solution where $R_{2k}(x)$ is defined by (2.1) with $\alpha = 2k$. Moreover, Tellez (1994) has proved that $R_{2k}(x)$ exists only for case p is odd with $p + q = n$.

In this paper we develop the equation (1.1) to the form

$$\sum_{r=0}^{m} c_r \Box^r u(x) = f(x), \tag{1.3}$$

which is called *the compound ultra-hyperbolic equation* and by convention $\Box^0 u(x) = u(x)$. We use the method of convolution of tempered distribution to find the solution of equation (1.3).